

## Estimation of Nonlinear Time-dependent Soil Behavior in Strong Ground Motion Based on Vertical Array Data

O. V. PAVLENKO<sup>1</sup> and K. IRIKURA<sup>2</sup>

*Abstract*—To improve our understanding of nonlinear elastic properties of soils, a method is proposed of estimation of stress-strain relations of soils *in situ* in strong ground motion based on vertical array data. Strong motion records provided by seismic vertical arrays allow estimation of nonlinear stress-strain relations in soil layers at different depths, from the surface down to the location of the deepest device. As an example, records obtained during the main shock of the 1995 Kobe earthquake at Port-Island, SGK, and TKS sites were used to estimate the stress-strain relations in the soil profiles. For different layers, different types of nonlinear stress-strain relations were selected, according to the profiling data. To account for temporal changes in the soil behavior, consecutive parts of records were examined, and for successive time intervals, the relations were found showing the best-fit approximation to the observed data. At Port Island and SGK sites, where the strongest accelerations were recorded, the obtained stress-strain relations showed systematic changes in the upper layers (0–14 m), such as, a progressive reduction of the slopes of the stress-strain curves due to liquefaction at Port Island and reduction and recovery of the slopes at SGK and TKS sites. At the three sites, the stress-strain relations remained stable in layers below 11–14 m. Thus, the proposed approach gives us a representation of the soil behavior in layers at different depths in strong ground motion; it allows calculation of the propagation of arbitrary seismic signals in the studied profiles and estimation of nonlinear components in the ground response by the nonlinear system identification technique. The method can also be applied to evaluate the ground response at sites where profiling data are available and an imposed motion can be estimated.

**Key words:** Strong ground motion, nonlinear soil behavior, liquefaction, stress-strain relations, seismic vertical arrays.

### *Introduction*

Recent earthquakes, such as the 1985 Michoacan earthquake, the 1989 Loma Prieta earthquake, the 1994 Northridge earthquake, and the 1995 Kobe earthquake provided new experimental data on the soil behavior in strong ground motion, in particular, on the liquefaction phenomena, and discussions on the nonlinearity of soil behavior were induced (LOMNITZ *et al.*, 1995; AGUIRRE and IRIKURA, 1997; FIELD *et al.*, 1997; O'CONNELL, 1999, etc.). Though nonlinear elastic properties of soils were

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<sup>1</sup> Institute of Physics of the Earth, Russian Academy of Sciences, B. Gruzinskaya 10, Moscow 123995, Russia. E-mail: olga@synapse.ru

<sup>2</sup> Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan. E-mail: irikura@egmdpri01.dpri.kyoto-u.ac.jp

studied in multiple laboratory experiments, and valuable laboratory experimental data are accumulated sometimes, this is not sufficient for understanding the soil behavior *in situ*, because soils often represent multicomponent systems containing water, air, gases, etc., and strong ground motion can induce movement and redistribution of these components, i.e., changes in the properties of the soils. Experimental data on the soil behavior in strong motion *in situ* are still few, fragmental, and non-representative; and accumulation of these data is important for improving our understanding of soil behavior in strong motion.

In strong ground motion Hooke's law does not hold for subsurface soils, i.e., soils should be taken as nonlinear systems transforming incident seismic signals into movement on the surface. For studying nonlinear properties of systems, effective methods are developed in system analysis, so-called nonlinear system identification technique, based on the determination of higher-order impulse characteristics of the systems. An output of a nonlinear system is represented as the Volterra-Wiener series, i.e., a sum of the response of a linear system to the input signal and a number of nonlinear corrections, which are due to quadratic, cubic nonlinearity, and nonlinearities of higher (the 4-th, 5-th, etc.) orders. If we know the input and output of a nonlinear system, we can judge regarding the types and quantitative characteristics of the system nonlinearity (MARMARELIS and MARMARELIS, 1978). Nonlinear identification of soils in various geotechnical conditions seems to be promising, because it allows a better understanding of the behavior of soils and structures in strong ground motion. However, to apply methods of system analysis to studying nonlinear properties of soils, knowledge of stress-strain relations in the soil layers in strong motions is required. In this paper, a method of estimation of nonlinear stress-strain relations in soils in strong ground motion is proposed based on vertical array data.

Numerous methods and programs developed for calculating the ground response in strong motion in various conditions do not allow estimation of stress-strain relations in soil layers *in situ*. Moreover, in cases of strong nonlinearity, there often remains some disagreement between the observed and simulated records. As is known, equivalent linear models (SHAKE, QUAD-4, FEADAM, LUSH, FLUSH) are not applicable for calculation of such complex phenomena as soil liquefaction. Programs DESRA (LEE and FINN, 1978), TARA and their modifications (FINN *et al.*, 1986; FINN and YOGENDRAKUMAR, 1989) allow determination of the possible level of the pore pressure and the possibility of liquefaction, and they can be applied for the analysis of soil behavior after liquefaction. Changes in the pore pressure are related to the volumetric deformations in soils in drained conditions, and one-dimensional diffusion is included in the algorithm. Programs DYSAC2, DYNAFLOW, and SWANDYNE are considered to provide the best results (ARULANANDAN *et al.*, 1995). Equations of motion of the liquid and solid phases are related to the equation of conservation of matter. Generation and dissipation of the pore pressure are connected with the deformation of the solid matrix due to the Biot equations (BIOT, 1956). However, in every

case simplifications and assumptions are applied, concerning the medium properties, as well as the mechanisms of the processes, therefore, any uncertainties and mistakes in modelling lead to an improper calculation of the soil movement.

At the same time, records of strong ground motion provided by seismic vertical arrays allow estimation of stress-strain relations in soil layers *in situ*. This paper describes the method of estimation of stress-strain relations and its application to the 1995 Kobe earthquake. The method allows us to trace temporal changes in the stress-strain relations. Since the estimates are based only on real measurements, they are free of theoretical approximations and physical assumptions concerning mechanisms of processes arising in the medium in strong ground motion.

### *Data and Method*

Data were processed for three recording sites, Port-Island, SGK, and TKS. Distances to the closest point on the fault line are 2 km, 6 km, and 16 km, respectively. Figure 1, taken from the paper by SATO *et al.* (1996), shows the locations of the sites, the major principal axes, and the epicentres of the main shock and aftershocks summarised by Disaster Prevention Research Institute of Kyoto University.

At Port Island, the vertical array contains four three-component accelerometers, installed at GL-0 m, GL-16 m, GL-32 m, and GL-83 m; the arrays at SGK and TKS sites consist of three three-component devices at GL-0 m, GL-24.9 m, and GL-97 m, and GL-0 m, GL-25 m, and GL-100 m, respectively (Fig. 2). We checked the directional drifts of the accelerometers by calculating the horizontal orbits of the long-period particle motions for the main shock and the aftershocks at different depths at the three sites. At Port Island, N19°W rotation at GL-83 m was detected and corrected; at SGK site, a reverse of NS component and N6°W rotation at GL-24.9 m and a reverse of NS component and N34°E rotation at GL-97 m were detected and corrected. At TKS site, N23°W rotation at GL-25 m and N9°E rotation at GL-100 m were detected and corrected. All these corrections agree with the conclusions of other authors.

The material at the three sites are similar to one another: reclaimed soil, clays, sands, and gravel (Fig. 2). The profiling data at Port Island used for nonlinear simulation were taken from (AGUIRRE and IRIKURA, 1997). Shear wave velocity, shear modulus degradation, and maximum shear strain at SGK and TKS sites were taken from SOEDA *et al.* (1999). These data were used to calculate the shear stress in failure  $\tau_{\max}$  and the low-strain shear modulus  $G_{\max}$  in different layers, following SEED *et al.* (1984) and SUN *et al.* (1988). *P*-wave velocities, when exceeding  $\sim 1000$  m/s, indicate saturation of soils with water. The underground water level lay at about 13 m at Port Island and  $\sim 3$ –5 m at SGK and TKS. The deepest layers were Pleistocene gravelly soils and the upper layers consist of alternating Pleistocene gravel/clay layers, Holocene sand/clay layers and fill (SATO *et al.*, 1996).

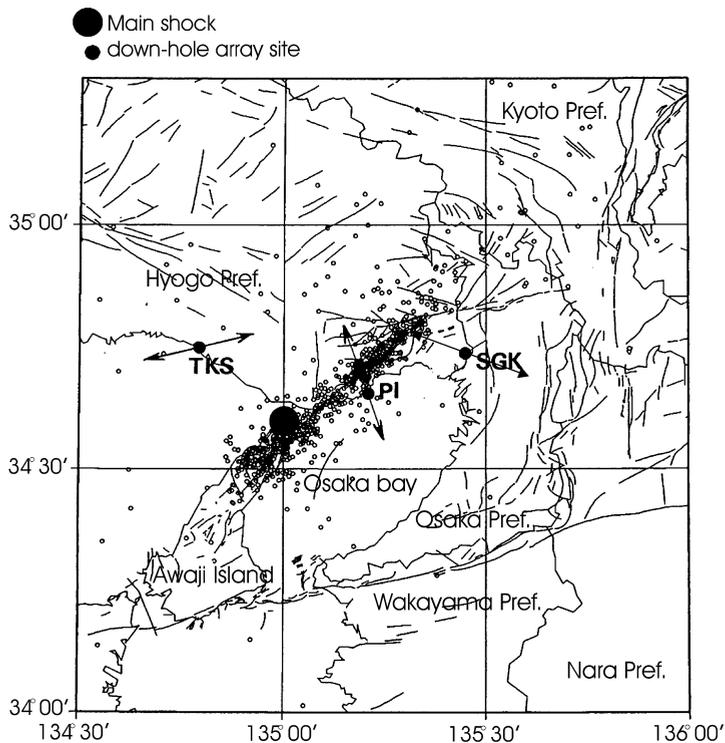


Figure 1

Locations of vertical array sites Port Island (PI), SGK, and TKS around Osaka bay, the major principal axes, and the epicentres of the main shock and aftershocks (derived from SATO *et al.*, 1996).

To estimate the stress-strain relations in different layers, vertical array records of strong motion and the profiling data were used. We applied the modified algorithm of calculation of the propagation of a vertically incident shear wave in soil layers up to the surface, based on the lumped mass method (JOYNER and CHEN, 1975), and combined it with the method of “trial and error” to find the stress-strain relations showing the best agreement between the observed and simulated records. In our computations, stress and strain are normalised in the manner used by HARDIN and DRNEVICH (1972): stress is normalised by multiplying by  $1/\tau_{\max}$ , and strain is normalised by multiplying by  $G_{\max}/\tau_{\max}$ . For calculations, the studied medium from the surface down to the location of the deepest device was divided into groups of layers, for which certain types of stress-strain relations were assumed (Fig. 2). Three main types of stress-strain curves were considered:

- (1) Those that are similar to laboratory experiments by HARDIN and DRNEVICH (1972), to describe the behavior of dense soils at depths;
- (2) Those of “soft” type, similar to type (1), but with greater slope, being close to horizontal for large strains, for liquefied soils;

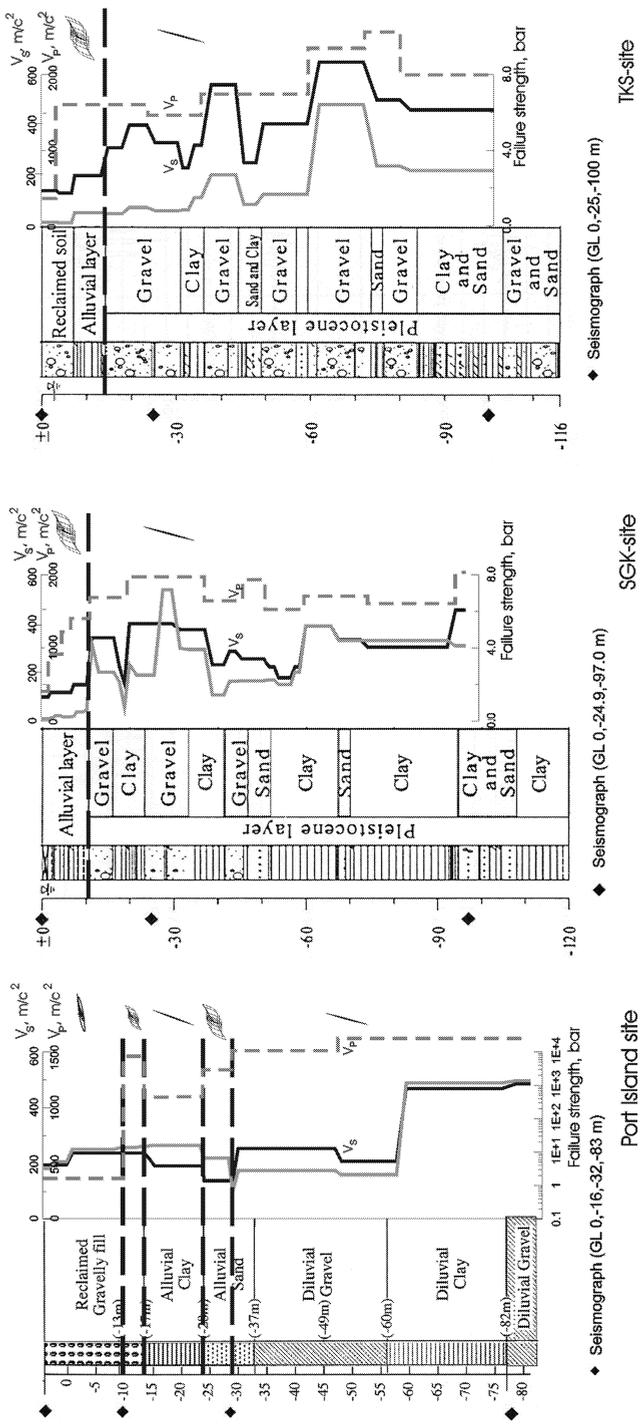


Figure 2  
The ground profiles and characteristic types of stress-strain relations at different depths at Port Island, SGK, and TKS sites.

(3) Those of “hard” type, declining to the stress axis at large strains, for water-saturated soils (terms “soft” and “hard” type stress-strain curves were introduced by ZVOLINSKII, 1982).

Sets of such curves were generated, and item-by-item examination was applied to find groups of curves showing the best-fit approximation to the observed data.

To account for temporal changes in the soil behavior, the records were divided into intervals of 1.5-seconds duration. Within each interval, the stress-strain relations were assumed to be stationary, and vary for different intervals. Calculations were performed (i.e., the “best-fit” stress-strain relations in the layers were determined) successively, interval by interval. No discontinuities occur at the boundaries of the intervals, because, in the next interval, the whole cycle of loading (or unloading) was recalculated for the new curve from its beginning. Only horizontal components of records were processed.

### *Results and Discussion*

The results of the simulation with the “best-fit” stress-strain relations show a good agreement between the observed and simulated records at the three sites.

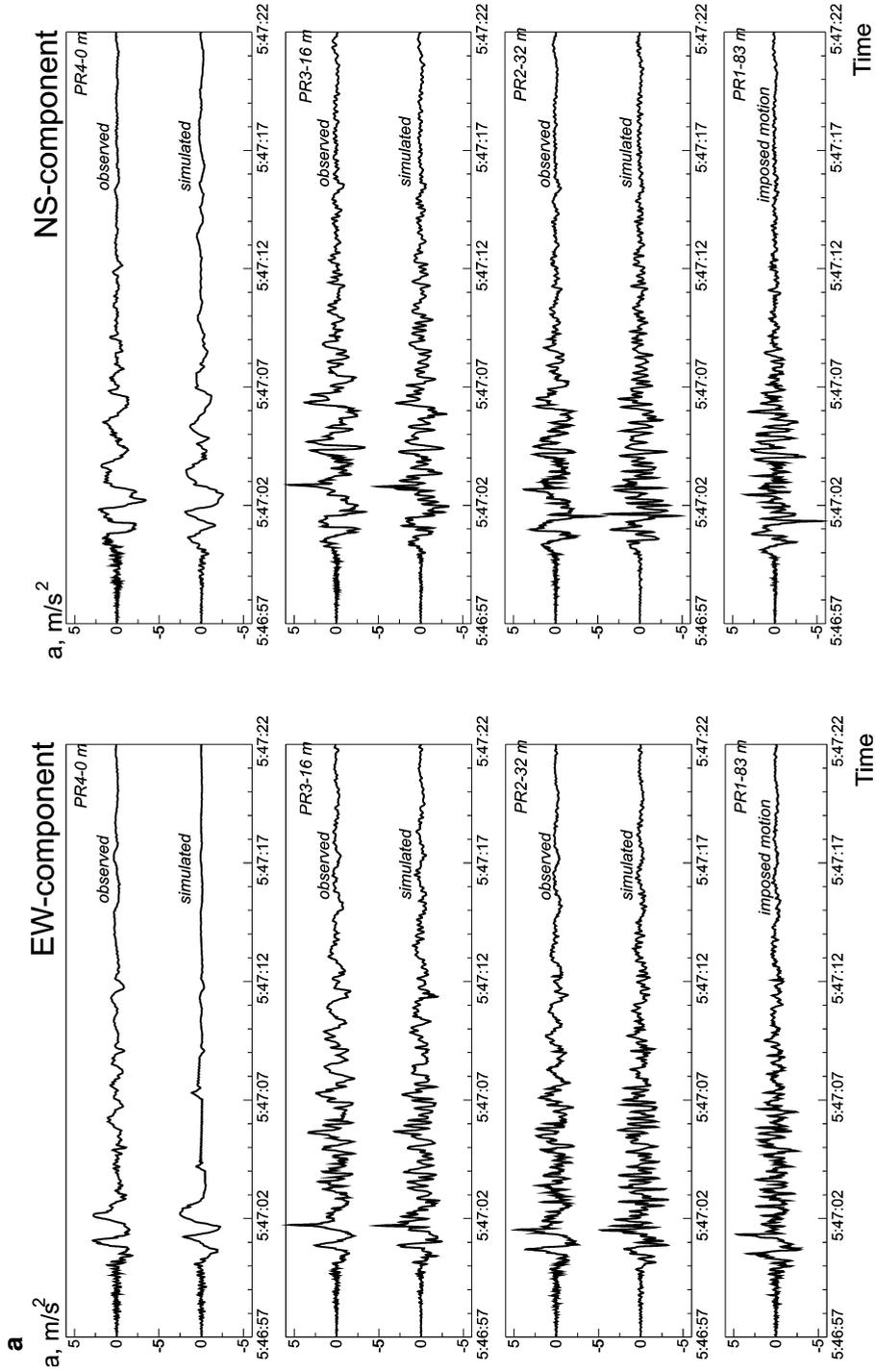
The most intense and complex movements and liquefaction were observed at Port Island. The strongest horizontal accelerations were measured at the deepest point of 83 m, such as,  $\sim 500$  Gal for the EW component and  $\sim 600$  Gal for the NS component. According to the material structure and the results obtained in the previous analysis (KAWASE *et al.*, 1995), the medium was divided into five groups of layers, for which certain types of stress-strain relations were assumed (Fig. 2). Since liquefaction occurred in the upper layers, curves of type (2) were applied for surface soils above the level of the underground water (0–13 m). Soil layers at depths 13–18 m and 27–32.5 m are reclaimed gravely fill and alluvial sand saturated with water, therefore, curves of type (3) were used to describe the soil behavior in these layers (Fig. 2). Peaks, or sharp increases in the amplitudes of the observed records at depths of 16 m and 32 m confirm that curves of type (3) are relevant in this case. Stress-strain curves of type (1), which are considered to be typical for dense soils at depth, were assumed for alluvial clay layers at 18–27 m and for deep diluvial layers below 32.5 m.

For successive 1.5-second time intervals, the groups of stress-strain relations were found, showing the best-fit approximation to the observed records. Figure 3 shows the observed and simulated records (a) and the obtained stress-strain relations at different depths (b) for ten successive time intervals.



Figure 3

The acceleration time history of the main shock in Port Island, observed and simulated (a), and the obtained stress-strain relations changing with time (b). The axes scales of the stress-strain relations are in relative units; the same for all time intervals at a given depth.





The most noticeable changes in the soil behavior were observed in layers near the surface, in the upper 13 m: the stress-strain relations become more and more sloping with time, showing a substantial progressive reduction of the shear modulus and liquefaction. At depths 13–18 m and 27–32.5 m, the obtained stress-strain curves show a slight reduction and the following recovery of the shear modulus. Below 32.5 m, no changes in the soil behavior are observed (Fig. 3b).

At SGK and TKS sites no liquefaction occurred, though sand boils observed after the quake around the TKS site indicate that liquefaction took place in the vicinities of this site.

Maximum accelerations recorded at SGK site were also high (Fig. 4a), up to 650 Gal on the surface for the EW-component. At this site, the soil behavior can be described by a relatively simple model: the soil profile is divided into two groups of layers, such as the near-surface alluvial layers (0–11 m), and the deeper layers (below 11 m), which are mostly gravel and clays. High values of *P*-wave velocities in the upper layers indicate the presence of water (Fig. 2), therefore, curves of type (3) are appropriate for the upper layers. Curves of type (1) were used for the deeper layers. Figure 4 represents the observed and simulated accelerograms (a) and the obtained stress-strain relations (b) for SGK site. The behavior of the layers below 11 m is stationary, whereas the stress-strain relations describing the behavior of the upper layers change with time: slopes of the curves decrease, then increase again, indicating reduction and recovery of the shear modulus in the upper layers.

At TKS site, maximum recorded accelerations were about 200 Gal, and the soil profile is represented by a water-saturated reclaimed fill and an alluvial layer in the upper part (0–14 m), and by gravel, clays, and sands in the deeper parts. Since the soil conditions are similar to that at SGK site, the choice of curves was also similar: curves of type (3) were selected for the upper 14 m, and curves of type (1) were chosen for the deeper parts. The observed and simulated accelerograms and the obtained stress-strain relations are shown in Figures 5a,b. The most intense movements took place in the layers below the level of the underground water, at depths 4–7 m. At TKS site, the soil behavior in the upper layers changes with time similar to SGK site: we observe reduction and the following recovery of the shear modulus, whereas, the behavior of layers below 14 m is stationary.

The obtained stress-strain relations seem to represent a fairly good approximation to reality. On one hand, they show a good agreement between the observed and simulated data. On the other hand, they give a description of the process which is physically correct, i.e., the stress-strain relations obtained for Port Island show progressive liquefaction in the upper layers and a stable behavior in the deeper parts. Reduction and a following recovery of the shear modulus is obtained for the upper layers at SGK and TKS sites. Similarity of the stress-strain relations obtained for two horizontal components is an additional factor testifying to the validity of the solution.

As shown above, the choice of the types of stress-strain relations in layers was physically justified. To check the possibility of different representations (i.e., vertical

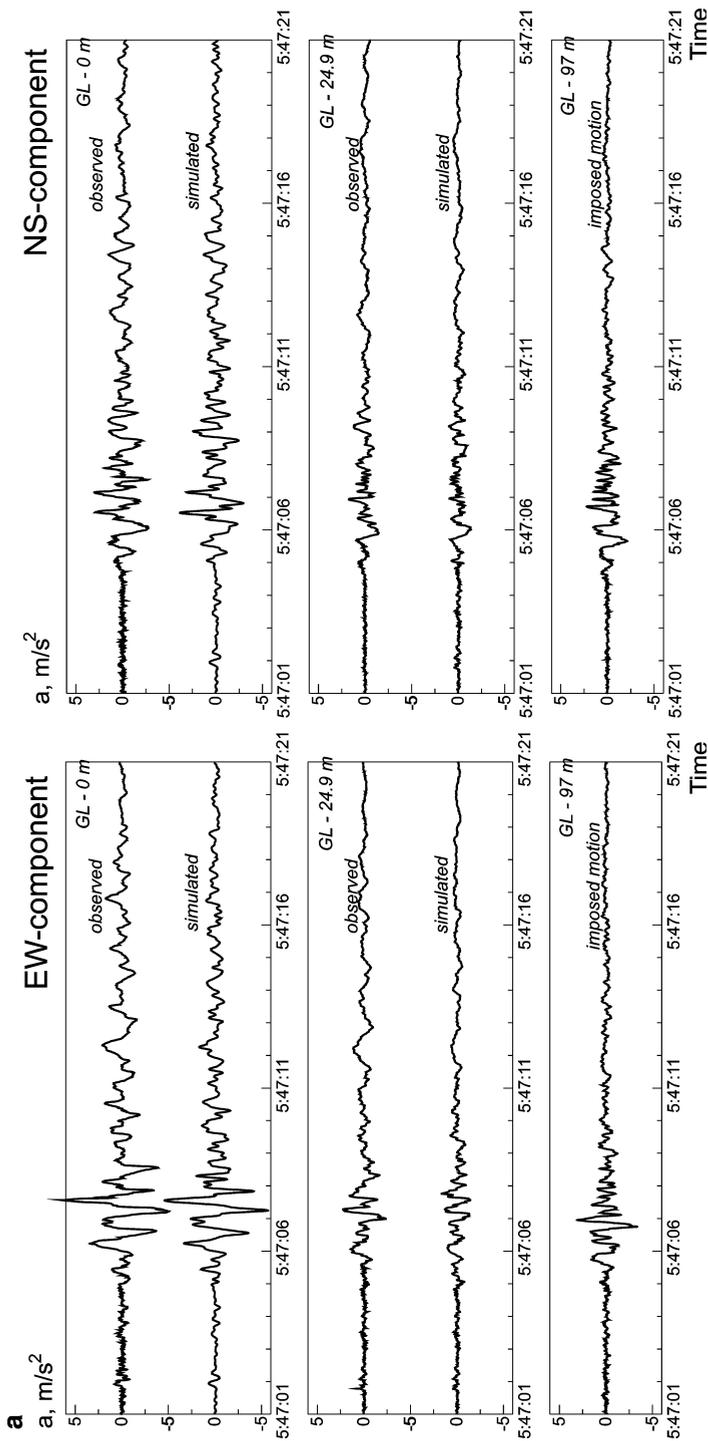


Figure 4

The acceleration time history of the main shock at SGK site, observed and simulated (a), and the obtained stress-strain relations changing with time (b). The axes scales of the stress-strain relations are in relative units; the same for all time intervals at a given depth.



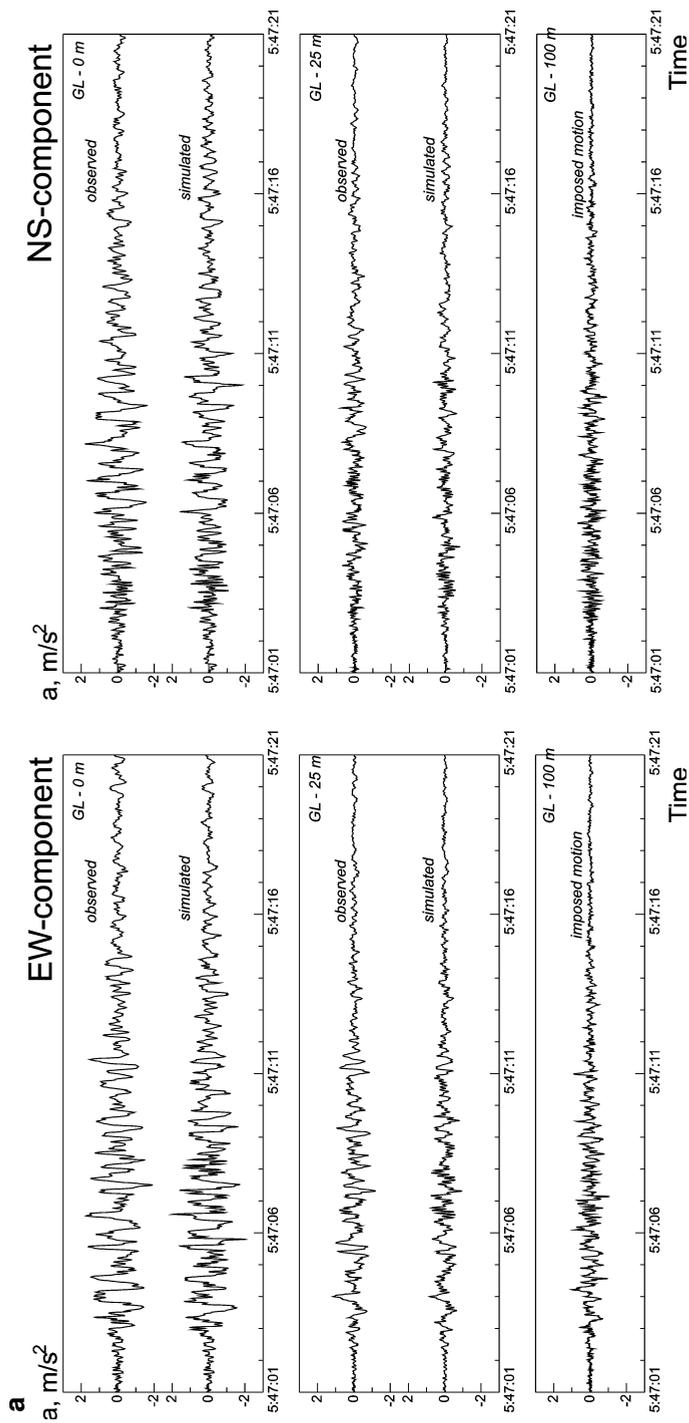


Figure 5

The acceleration time history of the main shock at TKS site, observed and simulated (a), and the obtained stress-strain relations (b). The axes scales of the stress-strain relations are in relative units; the same for all time intervals at a given depth.

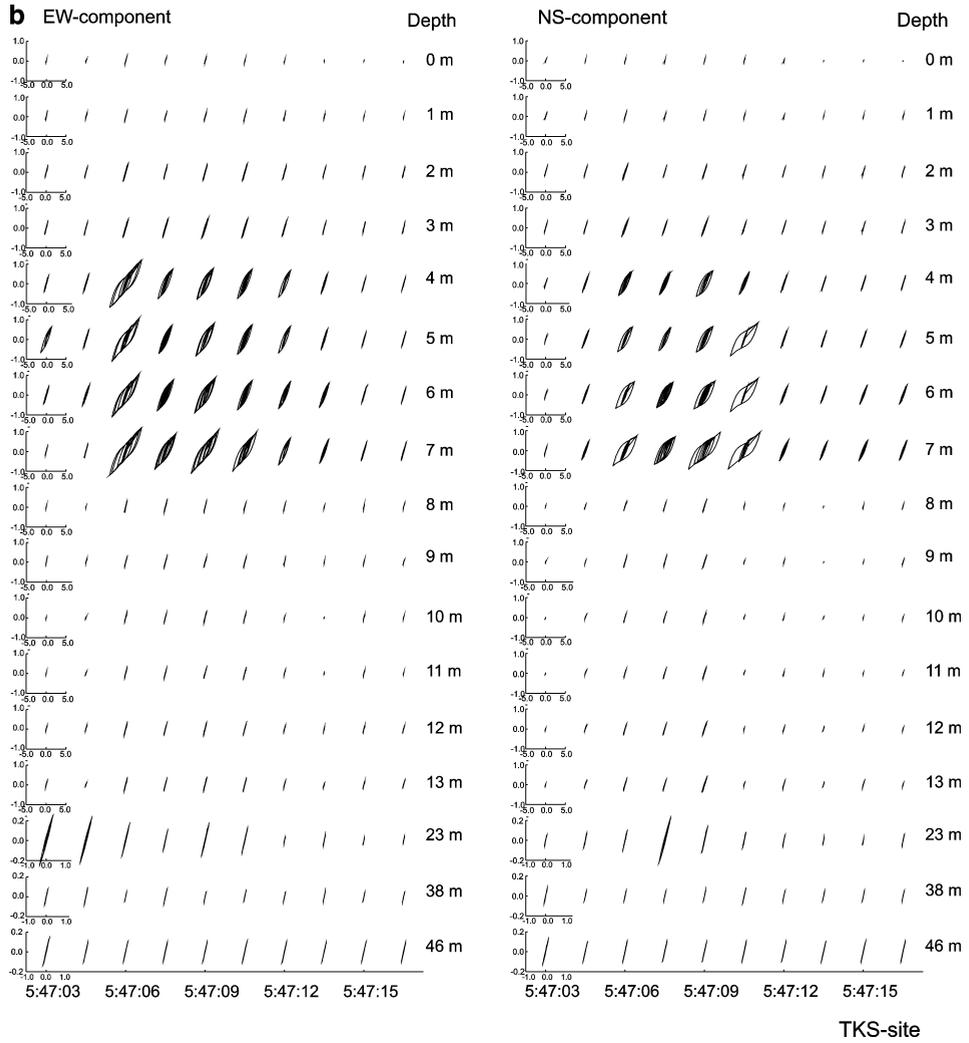


Figure 5  
(Contd.)

devices at SGK and TKS, and allow a more detailed description of the soil behavior in the layers.

*Conclusions*

Thus, time-dependent models of nonlinear soil behavior in the 1995 Kobe earthquake are constructed for three vertical array recording sites at Port Island, SGK, and TKS. The obtained stress-strain relations indicate temporal changes in the

soil behavior in the upper layers and a stationarity in the deeper parts. At Port Island, where the strongest horizontal accelerations were measured ( $\sim 700$  Gal at 83 m and  $\sim 340$  Gal on the surface), the most complicated vertical distribution of stress-strain relations was observed.

For weaker motions at SGK ( $\sim 300$  Gal at 97 m and  $\sim 650$  Gal on the surface) and TKS ( $\sim 100$  Gal at 100 m and  $\sim 200$  Gal on the surface), simpler behavior was observed. The medium structures are similar at these three sites, therefore, differences in the soil behavior are due to the differences in the acceleration amplitudes of the input signals. The soil behavior shows increased complexity as amplitudes become greater. Underground water plays an important role, since it influences soil behavior in strong motion and the possibility of liquefaction.

Our results confirm that in strong ground motion, soils not only change the dynamic parameters of the propagating seismic waves, but they also substantially change their properties, and these phenomena are inseparable from each other.

Our proposed method of estimating the stress-strain relations in soil layers at vertical array sites improves our understanding of processes arising in soils subjected to strong motion. Knowledge of stress-strain relations in soil layers allows us to perform further studies by calculating the propagation of seismic signals of various types in the soil profiles. We can calculate the propagation of the varieties of imposed motions—to estimate a range of possible ground responses; the propagation of monochromatic signals—to investigate the generation of higher harmonics, and the propagation of the Gaussian white noise—to apply the nonlinear system identification technique and determine the types and quantitative characteristics of the nonlinearity of the system of soil layers.

Since the selected “best-fit” curves are determined by the profiling data, the method allows constructing models of soil behavior in strong motions in various geotechnical conditions and gives us knowledge, which can be useful in predicting ground motion in future earthquakes. The method can be applied in any site where profiling data are available and an imposed motion can be estimated.

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