Types of elastic nonlinearity of sedimentary soils

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Abstract. Vertical array records of the 1995 Kobe earthquake showed clearly nonlinearity of soil behavior. There have been few observational validations, although, scientific publications on soil nonlinearity appeared since 1970's. In theoretical studies, nonlinearity has been introduced in constitutive relations as the quadratic correction to Hooke's law, however, the question of the types of soil nonlinearity was not investigated. Here we show, on the basis of data processing and nonlinear system identification technique, that sediments represent physical systems possessing mostly odd types of nonlinearity. Even-order nonlinearities become comparable with odd-order ones only in special cases, like liquefied soils. Though even and odd nonlinearities have much in common, frequencies of combination harmonics, shapes of seismic solitary waves, and other nonlinear phenomena are different in these two cases. On the whole, nonlinearity of soils leads to changes in spectra of propagating seismic signals: the spectra tend to take the form of $E(f)\sim f^{-n}$.

1. Physical mechanism of soil nonlinearity – deviations from Hooke's law

Nonlinear behavior of soils in strong ground motions includes different phenomena: changes in amplitudes and spectra of oscillations depending on ground conditions, soil consolidation or loosening, liquefaction, residual deformations, etc., which are known since 1930's [*Suyehiro*, 1932]. Representative experimental data on soil nonlinearity were obtained by the Lotung vertical array in Taiwan and by vertical arrays in Japan during the 1995 Kobe earthquake. In theoretical studies on soil nonlinearity, which appeared since 1970's, nonlinearity has been usually introduced into constitutive relations as the quadratic correction to Hooke's law [e.g., *Aleshin et al.*, 1981; *Gushchin and Shalashov*, 1981; *Lund*, 1983; *Parker*, 1988; *Dimitriu*, 1990; *Groshkov et al.*, 1990; *Nikolaev et al.*, 1995]; the question of the types of soil nonlinearity has not been investigated.

To investigate the types of soil nonlinearity, let us consider the physical cause of soil nonlinearity, i.e., nonlinearity of the stress-strain relationships of soils in strong ground motions. In strong motion, Hooke's law does not hold for sediments, and nonlinear corrections of higher-orders become significad

$$+\int_0^\infty\int_0^\infty k_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 +$$

$$+\int_0^\infty\int_0^\infty\int_0^\infty k_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 +...$$

(1)

where $x(t)$ is an input, $y(t)$ is the output, t is time, $\tau, \tau_1, \tau_2, \tau_3$ are time delays, k_0, $k_1(\tau)$, $k_2(\tau_1,\tau_2)$, $k_3(\tau_1,\tau_2,\tau_3)$ are the zero-order, first-order, second-order, and third-order Volterra kernels of the system. The first-order kernel $k_1(\tau)$ describes the linear part of the system response, while other terms represent quadratic, cubic and other nonlinear corrections. Thus, analysing input and output of a nonlinear system, we can judge about the types and quantitative characteristics of the system nonlinearity [*Marmarelis and Marmarelis*, 1978].

As is known, real stress-strain relationships of soils are very diverse and depend on the granulometric composition of a soil, its humidity, etc. In water-saturated soils, the initial convex-up part of the stress-strain curve is followed by a deviation to the stress axis, i.e., concave-up part. In such soils, we can expect shock waves beyond the limit of elasticity [*Zvolinskii,* 1982]. Stress-strain relations represented by the only convex-up parts are characteristic for soft soils like loess loams and sands. Liquefied soils can be described by similar "soft"-type stress-strain curves, but more sloping, with slopes close to horizontal at large strains. Sensitivity of soils to dynamic loading obviously decreases with depth, and soils at depths behaive "more linearly" than subsurface soils.

2. Estimation of stress-strain relationships of soils *in situ* using vertical array records

Based on these physical representations, we investigated records of the 1995 Kobe earthquake and its aftershocks provided by vertical arrays at Port Island (PI), SGK, and TKS sites, in order to estimate the stress-strain relations *in situ* [*Pavlenko and Irikura*, 2001,a]. For calculations, the soil profiles were divided into groups of layers, for which physically justified types of stress-strain relations were assumed (Figure 1). Sets of such curves were generated, and item-by-item examination was applied to find groups of curves showing the best-fit approximation between the the observed and simulated data. To account for temporal changes in soil behavior, successive 1.5-second time intervals were analysed. The calculation algorithm based on the modified lumped mass method [*Joyner and Chen*, 1975] was used. The obtained stress-strain relations (Figures 2,3,4) were used to trace changes in shear moduli in different layers [*Pavlenko and Irikura*, 2001,b]. At Port Island and SGK sites, where the strongest accelerations were recorded, maximum changes in upper layers and stability in deeper parts were obtained. A progressive reduction of the shear modulus due to liquefaction was observed at Port Island, and reduction and recovery of the shear modulus were noticed at SGK and TKS sites.

The obtained stress-strain relations (Figures 2,3,4) represent a fairly good approximation to the reality, because (1) the choice of stress-strain curves in layers was physically justified; (2) a good agreement was obtained between the observed and simulated records; (3) similarity of the stress-strain curves obtained for two horizontal components testifies to the validity of the solution; (4) the obtained changes in shear moduli are physically correct and agree with results obtained by other authors [*Kawase et al.*, 1995; *Sato et al.*, 1996] and with laboratory tests. Since the choice of curves in layers was physically justified, the obtained vertical distribution of the types of nonlinear behavior of soils is unique. Our calculations show that there is no other vertical distribution of stress-strain relations, which could give a similar good agreement between simulations and observations [Pavlenko and Irikura, 2001,ab].

3. Changes in spectra of signals propagating in soils, types of soil nonlinearity

Thus, time-dependent models of nonlinear soil behavior in strong ground motions are constructed for three recording sites. To determine the types and quantitative characteristics of the soil nonlinearity, the obtained models were tested by the Gaussian white noise (at 0.1 - 100 Hz) and by monochromatic signals (testing signals represented approximations to the imposed motion of the Kobe earthquake).

The results are shown in Figures 5 and 6. Functionals of the Wiener series (similar to Volterra series and consisting of terms, which are orthogonal with respect to the input signal in the form of the Gaussian white noise) were calculated [*Marmarelis and Marmarelis*, 1978], and linear, nonlinear quadratic, and nonlinear cubic components of the ground response were estimated for PI, SGK, and TKS sites (Figure 5). At PI, liquefaction occurred in the upper layers, and four time intervals are analysed, corresponding to different stages of liquefaction (Figure 5a). At SGK and TKS sites, similar results were obtained for successive time intervals, and examples are presented in Figure 5b.

As liquefaction developed at Port Island, the nonlinear part of the ground response increased, due to the increase of the quadratic and other even-order components, whereas the cubic component remained at the same level (Figure 5, a). Zero-order kernels h_0 represent quasi-static deformations of the surface. For different time intervals of the response, they take positive or negative values, i.e., surface layers slowly shift to one or the other side in their oscillations. The effect can be interpreted as a result of accumulation of residual shift deformations on the surface [*Zvolinskii*, 1982]. This asymmetry of oscillations is connected with even-order nonlinearities [*Marmarelis and Marmarelis*, 1978]. The intensity of these quasi-static deformations, normalized by the intensity of the response, increases, while liquefaction developes (Figure 5a), indicating an increase in the even-order nonlinearities.

Below in the figure, diagonal values of the Wiener kernels $h_1(\tau)$, $h_2(\tau_1,\tau_2)$, and $h_3(\tau_1,\tau_2,\tau_3)$ and the weighted-mean (weight being proportional to the maximum strain) stress-strain relations for the upper layers are presented. Loading and unloading parts of these relations can be represented as a Fourier-series, consisting of even (cosinusoidal) and odd (sinusoidal) parts, which are also shown in Figure 5a, b. The relations between these even and odd parts (square root of the ratio of their intensities is shown in Figure 5a, b) determine the relations of the even and odd components in the ground response. As liquefaction developes at PI, the stress-tsrain relations in the upper layers become more and more sloping, and the part of the even-order components in the ground response increases. For comparison, soils at SGK and TKS sites, where liquefaction did not occurr, possess mostly odd-order nonlinearities, which is evidently the most typical case for soils.

Another indicators of the types of soil nonlinearity are higher harmonics, generated during the propagation of monochromatic signals in soils. Figure 6 shows spectra of "input" monochromatic signals and spectra of simulated signals at depths of locations of recording devices at PI, SGK, and TKS. Generation of the 3d, 5th, 7th, and other odd-order harmonics testifyes to the odd-order nonlinearities in soils. Even-order harmonics (the 2d and 4th) are generated only in liquefied soils at PI. Thus, odd-order nonlinearities are typical for nonliquefied soils. Even-order nonlinearities can be detected only in soils, in which stress-strain relations contain noticeable even components, like in liquefied soils.

"Hysteretic effects" in rock were investigated analytically and in laboratory experiments [*Kadish et al.*, 1996; *Koen et al.*, 1996]. Hysteresis loops were approximated by parallelograms, and, if the parallelogram was narrow, the model predicted zero spectral density at even multiplies of the source frequency (which is in agreement with our conclusions) and approximate "pairing" of amplitudes for odd multiplies (which seems to be an artefact of the model).

Transformations of spectra of the Gaussian white noise propagating in soils show another important tendency: spectra of "output" signals on the surface tend to take the form of $E(f) \sim f^{-n}$ (n depends on the properties of soils) [*Kadomtsev and Karpman*, 1971]. Interactions of spectral components of propagating signals lead to generation of combination-frequency harmonics in the low and high frequency ranges; amplitudes of combination harmonics being related to their frequencies [*Kadomtsev and Karpman*, 1971]. As a result, energy is redistributed over the spectral band, so that low-frequency components increase, shapr spectral peaks dissappear, and the resulting spectrum of the output tends to take the "limiting" form of $E(f) \sim f^{-n}$ (Figure 6a). This formula was obtained theoretically in the approximation of an infinite number (or chaotic phases) of interacting waves [*Kadomtsev and Karpman*, 1971]. Therefore, the "limiting" spectrum can be attained only in cases of a very strong soil nonlinearity (i.e., thick sedimentary layers and intense imposed motion in a wide frequency band), whereas in practice we observe "intermediate" cases and only the tendency of increasing low-frequency components and smoothing spectral peaks. Acceleration spectra of the 1995 Kobe earthquake are shown in Figure 6b, and the tendency is clearly seen. Physical mechanizms of these phenomena, which are known in geotechnical engineering as overdamping of high-frequency seismic waves in soils, is elastic nonlinearity of subsurface soils.

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Running Heads:

PAVLENKO AND IRIKURA: TYPES OF ELASTIC NONLINEARITY OF SOILS

Figure Captions:

Figure 1. Nonlinear system identification with the Gaussian white noise in a wide frequency band at Port Island, four stages of liquefaction.
Figure 2. Nonlinear system identification with the Gaussian white noise in a wide frequency band at SGK and TKS sites.
Figure 3. Velocity spectra of testing monochromatic and Gaussian-white-noise signals propagating in soils at Port Island, SGK, and TKS sites.
Figure 4. Acceleration spectra of the 1995 Kobe earthquake at Port Island, SGK, and TKS sites.

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